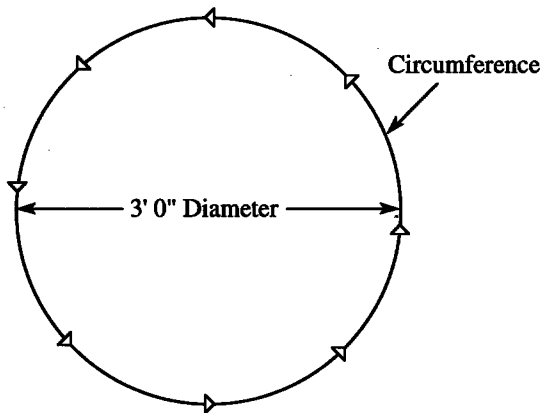


B

STUDENT'S GUIDE

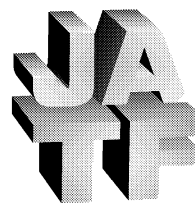
Basic Mathematics and Measurement

A Curriculum for Members of The
International Union of Painters
and Allied Trades



CIRCLE

International Union
of Painters and Allied Trades
Joint Apprenticeship and Training Fund
1-800-276-7289



*A Curriculum for the
members of the
International Union of
Painters and Allied Trades AFL-CIO*

The IUPAT JATF Educational Program, which involves this series of instructional materials in conjunction with classroom or correspondence-type instruction and on-the-job training, is provided to help maintain a constant supply of qualified workers for the industry. Everyone who enters the trade makes a definite commitment to themselves as well as to the employer. This includes a commitment to work diligently, to learn new techniques as well as improve on those already learned, to maintain an attitude which promotes learning, and to exercise a high degree of maturity in all matters related to the job.

These instructional materials are provided as a supplement to the on-the-job training everyone receives while performing their job. Every journeyman and apprentice must seek to learn from any source at their disposal in a continuing effort to enhance and upgrade their skills. The Industry is in a constant state of change and only through efficient and continued education will our trades remain state-of-the-art.

Copyright 1998
Revised October, 2000
by

**The International Union of Painters and Allied Trades
Joint Apprenticeship and Training Fund**
1750 New York Avenue NW, 8th Floor
United Unions Building
Washington, DC 20006

All rights reserved. This book, or parts thereof, may not be reproduced without written permission of the publisher.

Printed in U.S.A.

ACKNOWLEDGMENTS

We are extremely grateful to the following manufacturers and organizations for providing time, photographs, illustrations, and other materials for this book.

IUPAT JATF TRUSTEES

William S. Timmons, Co-Chairman, IUPAT General Vice President
Bob Swanson, Co-Chairman, President, Swanson and Youngdale, Inc., Minneapolis, MN
Lawrence Duck, President, Peak Commercial Painting Ltd., Vancouver B.C., CN
Charles E. Anderson, Business Manager, District Council #30
Robert Matson, Business Manager, District Council #5
Sandy Vagelatos, Business Manager, District Council #9
Bud Nicoll, President, West Valley Carpet Service, Inc., Milpitas, CA
Francis Wojehowski, President, Gypsum Specialists Contractors, Inc., St. Louis, MO
Joseph Glaab, Business Manager, District Council #711, Bloomfield, NJ
Alfred Wertz, Walters and Wolf Glass, Fremont, CA
Terry Webb, Eureka Glass Company, Philadelphia, PA
David Ottesen, IUPAT General President Representative, Seattle, WA
Roy Shoaf, Jr., Business Representative, District Council #3, Kansas City, MO
Bill Holsman, Sign-Lite Corporation, Cleveland, OH
Jim Watroba, Financial Secretary, Local Union #639, Cleveland, OH
Allan Delange, Kenny Manta Industrial Services, Hammond, IN

IUPAT JATF GLAZIER CURRICULUM COMMITTEE

Harry F. Schurr, Secretary, Apprentice Coordinator, Local Union #252
George Sandell, Training Coordinator, Local Union #1621
Douglas Melphy, Financial Secretary, Local Union #930
Mike Schuler, Training Coordinator, Local Union #188
Rudy Tasic, Apprentice Coordinator, Local Union #27
Richard Mauro, President, Tower Glass Co., Inc.
David Whitfield, Apprenticeship Instructor, Local Union #1044
Gilbert C. Humann, Business Manager, Local Union #188
Debi L. Humann, Technical Writer, Puget Sound Technical Writers
Michael Metz, Apprentice Coordinator, Local Union #25

IUPAT JOINT APPRENTICESHIP AND TRAINING FUND

Michael E. Monroe, General President
Richard Hackney, Administrator
Brian A. Gustine, Technical Assistance Coordinator

GEORGE MEANY CENTER FOR LABOR STUDIES INC.

Sue Schurman, Executive Director, George Meany Center
Chuck Hodell, Director, George Meany Center, Educational Design Unit
Julie Ann Mendez, Instructional Designer

OBJECTIVES

By the completion of this module the student should be able to:

- Demonstrate addition, subtraction, multiplication, and division skills.
- Demonstrate the general rules of English measurement.
- Identify denominate numbers.
- Demonstrate the “order of operation” when a problem involves multiplication, division, addition, and/or subtraction.
- Explain the metric system and how it can be used in the trade.
- Demonstrate how to find the surface area.

Mathematics is a basic and important tool. You will use simple mathematics daily in planning and executing your work. Mathematical accuracy is critical for a profitable job.

This topic begins with the basic facts of arithmetic and continues through some of the early stages of algebra. Many of us have areas in our mathematics background that are hazy, barely understood or troublesome. Reviewing these basic mathematical concepts may help you avoid making costly mathematical mistakes.

PLACE VALUE

In our base ten (decimal) system, digits and commas are the only symbols used. From right to left, the first three place value names are one, ten, and hundred as shown in Figure 1.

In the example, “573” the place values are as follows:

- 5 has a place value of hundred (100)
- 7 has a place value of ten (10)
- 3 has a place value of one (1)

In other words “573” is:

$$5 \text{ hundreds} + 7 \text{ tens} + 3 \text{ ones}$$

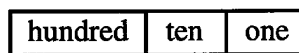


Figure 1 - Place Values

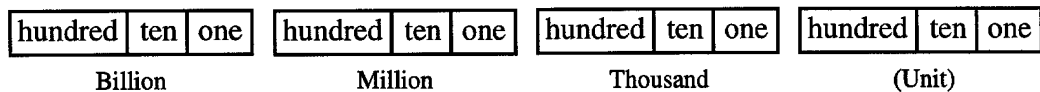


Figure 2 - Place Values

Continuing to the left, the digits are grouped in three (see Figure 2.) The first four groups are unit, thousand, million and billion.

The group on the far left may have one, two or three digits while all other groups **MUST** have three digits. In each group the names are the same: one, ten, hundred.

ADDITION AND SUBTRACTION

It is assumed that the reader knows basic addition and subtraction skills. The following will be a quick review.

The operation of addition is noted by the plus (+) sign. The numbers being added together are referred to as the “addends” and the answer is termed the “sum” or “total.”

Subtraction is noted by the minus (-) sign. Using the example of $9 - 3 = 6$, the “9” would be referred to as the “minuend,” the “3” is the “subtrahend,” and the answer is termed the “difference.”

Carrying Numbers

Addition problems having several addends usually produce sums greater than 9 in one or more of the columns (the ones column, the tens, the hundreds, etc.). When this occurs, the answer can be found by “carrying” (see Figure 3).

When carrying, the “extra” digits that are generated when a column sum exceeds 9 are carried over and added to the next column on the left. If, after adding, the answer is a single digit number, there is no need to carry.

Writing the problem with each addend grouped in terms of ones, tens, hundreds, etc. demonstrates more clearly the carrying process.

To carry the addend in Figure 3 perform the following steps.

$\begin{array}{r} 357 \\ 845 \\ + \underline{22} \end{array}$	$\begin{array}{r} 300 + 50 + 7 \\ 800 + 40 + 5 \\ \hline *** + 20 + 2 \\ 1100 + 110 + 14 \end{array}$	$\begin{array}{r} 357 \\ 845 \\ + \underline{22} \\ \hline 1224 \end{array}$
---	---	--

Figure 3 - Addition and Carrying

1. Write the numbers in columns so that place values are lined up. Add each column downward, starting with the ones. Add the 7, 5, and 2 to get a two digit sum of 14. Write down the 4, below the answer line in the ones column, and carry the 1 to the tens column.
2. Mentally add the 1 along with the other digits in the tens column, getting a sum of 12. Write down the 2 below the line in the tens column and carry the 1 to the hundreds column.

3. Mentally add the 1 along with the other digits in the hundreds column, getting a sum of 12. Write down the 2 in the hundreds column of the answer and carry the 1 to the thousands column.
4. If there were other digits in the thousands column to which the 1 could be added, the process would continue. Since there are no other digits to be added in the thousands column, this final 1 simply drops down and is written in the thousands place to complete the answer.

PRACTICE PROBLEMS

1. $686 + 240 + 1,320 + 16 + 400 =$
2. $16 + 480 + 26 + 15 + 6,000 =$
3. $29 + 16 + 24 + 13 + 10 =$
4. $18 + 24 + 32 + 1,001 + 216 =$
5. $290 + 1,360 + 4,510 + 6,216 + 3 =$

Borrowing Numbers

In subtraction, the borrowing process is the reverse of carrying. Borrowing must be done when the subtrahend in any one of the columns (ones, tens, hundreds, etc.) is larger than the minuend in the same column (see Figure 4).

$\begin{array}{r} 43 \\ - \underline{8} \end{array}$	$\begin{array}{r} 40 + 3 \\ - \underline{8} \end{array}$ <p>To check for accuracy: $35 + 8 = 43$</p>	$\begin{array}{r} 43 \\ - \underline{8} \\ \hline 35 \end{array}$
--	---	---

Figure 4 - Subtraction and Borrowing

To solve the problem in Figure 4, perform the following steps:

1. Write the numbers in columns so that the place values are lined up. Subtract each column, starting with the ones.
2. If the column cannot be subtracted, you must “borrow” from the column on the left. For example, since 8 cannot be subtracted from 3, a “ten”

must be borrowed and combined with the 3 in the ones column. Notice that borrowing a ten to increase the value of the digit in the ones column reduces the value of the digit in the tens column by 1.

3. 8 can now be subtracted from 13. Write down 5 under the answer line in the ones column. The 3 from the tens column can be brought down below the answer line to complete the equation with a difference of 35.
4. If it is necessary to borrow in more than one column, the borrowing process continues until all the subtrahends can be subtracted from the corresponding minuends to solve the problem.
5. To check for accuracy, add the difference to the subtrahend and the answer should equal the minuend as shown in Figure 4.

PRACTICE PROBLEMS

6. $280 - 116 =$
7. $40 - 16 =$
8. $1,226 - 456 =$
9. $126 - 14 + 34 =$
10. $973 - 792 =$

MULTIPLICATION

When two or more whole numbers are multiplied, each is called a “factor.” The answer is called the “product.” Multiplication may be indicated by the following:

- a multiplication sign (X)
- a dot between numbers
- or parentheses around the numbers to be multiplied.

$$6 \times 8 = 48$$

$$6 \cdot 8 = 48$$

$$6 (8) = 48$$

$$(6) (8) = 48$$

When a dot is used to indicate multiplication, the dot is placed above the line of writing distinguishing it from a decimal point or period. Notice also that when parentheses are used to indicate multiplication, the numbers to be multiplied are spaced closer together than they are when the dot or “x” is used.

An important part of multiplication is memorizing the multiplication tables. Multiplication tables must be learned so thoroughly that you have instant recall without hesitation or effort.

Another important property of multiplication involves the “zero product law.” This rule states that any number multiplied by zero equals zero. For example:

$$\begin{aligned}0 \times 0 &= 0 \\10 \times 0 &= 0 \\0 \times 140 &= 0\end{aligned}$$

Multiplying Whole Numbers

Multiplying whole numbers is a shortcut for repeated addition. For example: 6 (5) and 6×5 are read as six 5's. Of course, we would write 6 six times and add, however, multiplying to get the answer saves considerable time.

In the example of multiplying 6 times 27, we first arrange the factors in the following manner:

$$27 \times 6 = 162$$

This problem would be solved by performing the following steps:

1. 6 times 7 equals 42. The 2 is written down under the answer line in the ones column and the 4 is carried to the tens column.
2. 6 times 2 is 12. Add the 4 that was carried over from step 1 and write the result, 16, under the answer line as shown.
3. The final answer is 162.

When multiplying, it is important to place the digits of the factors in the proper columns. In other words, the ones must be placed in the ones column, tens in the tens column, etc. This is especially true when multiplying by a number that has more than two digits as shown in the following example:

$\begin{array}{r} 43 \\ x \underline{27} \end{array}$	$\begin{array}{r} 43 \\ x \underline{27} \\ 301 \end{array}$	$\begin{array}{r} 43 \\ x \underline{27} \\ 301 \\ \underline{860} \\ 1161 \end{array}$
---	--	---

To solve the above problem we would perform the following steps:

1. First multiply 7 times 3 which equals 21. The 1 is placed under the answer line in the ones column and the 2 is then carried over to the tens column. 7 times 40 equals 280 plus the 2 carried over from the ones column equals 300. 300 is recorded next to the one under the answer line.
2. Next we multiply 43 by 20. To begin, multiply 20 x 3 which equals 60. The 60 is placed under the answer line in the tens column. Next multiply 20 times 40 equaling 800 and record 800 in the hundreds place, as shown, in the corresponding column.
3. Add the products downward, keeping the numbers in the correct columns (ones, tens, etc.) The correct answer is 1,161.

PRACTICE PROBLEMS

PRACTICE PROBLEMS

11. $292 \times 16 =$
12. $292 \times 4 \times 3 + 2 =$
13. $320 \times 168 =$
14. A box of single strength window glass weighs 67 lbs. and a box of double strength window glass weighs 80 lbs. How many pounds will be left of a 16,280 lb. load if 16 boxes of single strength and 26 boxes of double strength are removed from the load? $=$
15. A steel sash job requires 1,200 lites of glass. Half of these lites have an area of more than 4 sq. ft. each. If two metal clips are required for each of the four sides of each piece having an area of 4 sq. ft. or less and if three metal clips are required for each of the four sides of each piece having an area of more than 4 sq.ft. how many metal clips are needed to do the complete job? $=$

DIVISION

Division is usually indicated either by a division sign (\div) or by placing one number over another number with a line between as shown in the following examples.

$$8 \div 4 = 2 \quad \text{or} \quad \frac{8}{4} = 2 \quad \text{or} \quad 4 \overline{)8} = 2$$

In this example the number 8 is the “dividend,” 4 is the “divisor” and 2 is the “quotient.”

Just as multiplication can be considered as repeated addition, division can be considered as repeated subtraction. For example, if we wish to divide 12 by 4, we may subtract 4 from 12 in successive steps and tally the number of times that the subtraction is performed, as follows:

$$\begin{array}{r} 12 \\ - \underline{4} \\ = 8 \\ - \underline{4} \\ = 4 \\ - \underline{4} \\ = 0 \end{array}$$

In this example, 4 was subtracted 3 times from 12. In other words, 4 is contained in 12 three times. Since successive subtraction is too cumbersome for rapid concise calculation, division is used as a quick, accurate replacement.

To solve the following problem, the following steps should be performed:

$4 \overline{)56}$	$\begin{array}{r} 14 \\ 4 \overline{)56} \\ \underline{40} \\ 16 \\ \underline{16} \\ 0 \end{array}$
--------------------	--

1. When dividing, begin with the far left column of the dividend. In this case, divide 4 into 50. 4 is contained in 50 ten times. Write 1 in the tens’s place over the 4 and write 40 as shown.
2. Subtract 40 from 56 for a difference of 16.
3. 4 divides into 16 four times. Write 4 in the one’s place over the 6 in the dividend and record 16 as shown. 16 subtracted from 16 equals 0.
4. The quotient for the above example is $56 \div 4 = 14$
5. To check the answer for accuracy, multiply the quotient times the divisor to equal the dividend. $14 \times 4 = 56$

To solve the following division problem, perform the following steps:

$7 \overline{)252}$	$\begin{array}{r} 36 \\ 7 \overline{)252} \\ \underline{210} \\ 42 \\ \underline{42} \\ 0 \end{array}$
---------------------	--

1. In some cases, the divisor (7) is too large to be contained in the first digit of the dividend (2). When this occurs, we divide the divisor into whatever number of digits in the dividend that allows the divisor to be divided. In this instance, we will divide the 7 into 250 (the first two digits of the dividend).
2. 7 is contained 30 times in 250. We write 3 along the tens column of the dividend. Multiplying 30 times 7 is 210. We write 210 below the dividend.
3. Subtracting 252 minus 210 is 42. We write down the 42 as shown in the example. We have now formed a new dividend, 42.
4. 7 is contained 6 times in 42. We write 6 above the ones column as shown. Multiplying as before, 6 times 7 is 42. We write the product as shown in the example. Perform the remaining subtraction: $42 - 42 = 0$
5. The final quotient is $252 \div 7 = 36$. To check this answer for accuracy, multiply the answer times the divisor: $36 \times 7 = 252$.

Uneven Division

Uneven division refers to when the dividend cannot be divided evenly into the divisor. The leftover numbers are called the remainders and they are placed next to the quotient with the prefix "R."

$12 \overline{)186}$	$\begin{array}{r} 15 \text{ R}6 \\ 12 \overline{)186} \\ \underline{120} \\ 66 \\ \underline{60} \\ 6 \end{array}$
----------------------	--

1. 12 is contained in 180 ten times. 1 is written over the 8 in the tens column. Multiply 10 times 12 for an answer of 120. Record the 120 as shown. Subtracting 120 from 186 for a difference of 66.
2. 12 divides into 66 five times. 5 times 12 equals 60. The 5 is written over the 6 in the ones column. Record the 60 as shown and subtract 60 from 66 for a difference of 6.
3. Since 12 cannot be divided into 6 the six is recorded with the answer following the "R" which stands for remainder.
4. 186 divided by 12 equals 15 with a remainder of 6.

5. To check the answer for accuracy, multiply the quotient by the divisor and add the remainder to equal the dividend. For example:

$$15 \times 12 + 6 = 186$$

PRACTICE PROBLEMS

16. $240 \div 6 =$
17. $180 \div 5 =$
18. $64 \div 8 =$
19. $351 \div 13 =$
20. $38,250 \div 15 =$

DENOMINATE NUMBERS

Numbers that have a unit of measurement associated with them, such as yard, kilowatt, pound, pint, etc., are denominate numbers. The word “denominate” means that the numbers have been given a name: they are not just abstract symbols.

To add, subtract, multiply or divide denominate numbers only like units or units of the same kind can be combined. For example, the sum of 1 foot and 3 inches would not be 4 feet nor is it 4 inches. Since only like units can be combined, the correct answer would be 1 foot 3 inches.

If denominate numbers are to be combined and the units are not alike, equivalent measurements must be found. Equivalent measurements refer to measurements of the same thing only of the amount using a different unit or name. For example, 12 inches is an equivalent measurement of 1 foot.

To change the unit of a measurement, replace the unit of measure with an equivalent measurement and multiply, divide, subtract or add as necessary. For example:

$$2 \text{ yards } 2 \text{ feet } 11 \text{ inches} = \underline{\hspace{2cm}} \text{ inches}$$

$$72 \text{ inches} + 24 \text{ inches} + 11 \text{ inches} = 107 \text{ inches}$$

and

$$3 \text{ quarts } 1 \text{ pint} = \underline{\hspace{2cm}} \text{ pints}$$

$$6 \text{ pints} + 1 \text{ pint} = 7 \text{ pints}$$

Also, whenever possible the final answer should be in the simplest form. To add denominate numbers (as shown in Figure 5A), perform the following steps:

1. Add 8 inches and 5 inches together recording the answer of 13 inches below the answer line as shown.
2. Add 6 feet and 4 feet, recording the answer of 10 feet below the answer line.
3. Since 13 inches is the equivalent of 1 foot and 1 inch, we simplify or regroup the answer to 11 feet 1 inch.

A similar example of adding denominate numbers occurs when reading a transit (see Figure 5B). To add degrees, minutes and seconds perform the following steps:

1. 6 seconds added to 5 seconds equals 11 seconds. The answer is written below the answer line under the seconds column.
2. 44 minutes plus 22 minutes equals 66 minutes. Since 60 minutes make one hour, the 1 is carried over to the hour column and 6 minutes is recorded under the answer line in the minutes' column. (66 minutes - 60 minutes = 6 minutes)
3. 20 hours, 13 hours and the 1 carried over hour are added together for answer of 34 hours.

Figure 5

A)

$$\begin{array}{r}
 6 \text{ feet } 8 \text{ inches} \\
 + 4 \text{ feet } 5 \text{ inches} \\
 \hline
 10 \text{ feet } 13 \text{ inches}
 \end{array}$$

or, in its simplest form:

$$11 \text{ feet } 1 \text{ inch}$$

*** *** ***

B)

$$\begin{array}{r}
 20 \text{ hours } 44 \text{ minutes } 6 \text{ seconds} \\
 + 13 \text{ hours } 22 \text{ minutes } 5 \text{ seconds} \\
 \hline
 33 \text{ hours } 66 \text{ minutes } 11 \text{ seconds}
 \end{array}$$

or, in its simplest form:

$$34 \text{ hours } 6 \text{ minutes } 11 \text{ seconds}$$

Subtracting Denominate Numbers

Subtracting denominate numbers involves regrouping like groups. For example, if we wanted to subtract the following denominate numbers we would have:

$$\begin{array}{r} 28 \text{ hours} \quad 4 \text{ minutes} \quad 3 \text{ seconds} \\ - 16 \text{ hours} \quad 8 \text{ minutes} \quad 2 \text{ seconds} \end{array}$$

In order to subtract 8 minutes from 4 minutes we would need to regroup (borrowing from the hour column as follows:

$$\begin{array}{r} 27 \text{ hours} \quad 64 \text{ minutes} \quad 3 \text{ seconds} \\ - 16 \text{ hours} \quad 8 \text{ minutes} \quad 2 \text{ seconds} \\ \hline 11 \text{ hours} \quad 56 \text{ minutes} \quad 1 \text{ second} \end{array}$$

Multiplying Denominate Numbers

When one denominate number is multiplied by another the product or answer becomes a square unit. For example, 1 foot times 1 foot equals one square foot.

If it is necessary to multiply combined numbers, such as 2 yards 1 foot x 6 yards 2 feet, the units will need to be converted to like units (such as fractions of a yard or all the units to feet).

To multiply denominate numbers, using the following example, proceed as follows:

$$\begin{array}{r} 5 \text{ yards} \quad 2 \text{ feet} \quad 6 \text{ inches} \\ \times 3 \\ \hline \end{array}$$

1. Since only like units can be multiplied together then each unit should be multiplied by 3 as shown:

$$\begin{array}{l} 3 (5 \text{ yards}) = 15 \text{ yards} \\ 3 (2 \text{ feet}) = 6 \text{ feet} \\ 3 (6 \text{ inches}) = 18 \text{ inches} \end{array}$$

2. After multiplying all units, reduce the answer to its simplest form as shown:

$$\begin{array}{r} 15 \text{ yards} \quad 6 \text{ feet} \quad 18 \text{ inches} \\ \text{TO} \\ 17 \text{ yards} \quad 1 \text{ foot} \quad 6 \text{ inches} \end{array}$$

Division of Denominate Numbers

When dividing denominate numbers, start with the largest unit first. If there is a remainder, convert it to the next lower unit. Repeat the division process until all units have been divided as shown in the following example:

$$24 \text{ gallons } 1 \text{ quart } 1 \text{ pint} \div 5 =$$

1. Divide the 24 gallons by 5

$$\begin{array}{r} 4 \\ 5 \overline{)24} \text{ gallons} \\ \underline{20} \text{ gallons} \\ 4 \text{ gallons (remainder)} \end{array}$$

2. Convert the 4 remaining gallons to 16 quarts and add them to the 1 quart from the original problem. Divide the 17 quarts by 5 as shown:

$$\begin{array}{r} 3 \\ 5 \overline{)17} \text{ quarts} \\ \underline{15} \text{ quarts} \\ 2 \text{ quarts (remainder)} \end{array}$$

3. Convert the remaining 2 quarts to 4 pints and add to the 1 pint from the original problem. Divide 5 pints by 5 as shown:

$$\begin{array}{r} 1 \\ 5 \overline{)5} \text{ pints} \\ \underline{5} \text{ pints} \\ 0 \end{array}$$

4. The final quotient is:

$$24 \text{ gallons } 1 \text{ quart } 1 \text{ pint} \div 5 = 4 \text{ gallons } 3 \text{ quart } 1 \text{ pint.}$$

PRACTICE PROBLEMS:

21. 1 ft 3 inches + 3 ft. 9 inches =
22. If a piece of metal 2 ft. 5 in. long is cut from a piece that is 8 ft. 3 in. long, how much metal will be left over?
23. 5 minutes 30 seconds - 3 minutes 42 seconds =
24. What is the total length of the following pieces of metal: 1 piece 1 ft 10 in. long; 1 piece 10 ft. 2 in. long; 1 piece 16 ft. 3 in. long; and 1 piece 8 ft. 1 in. long?
25. A mitered plate glass store front needs recementing. It includes 4 joints: 10 ft 6 in. long, 2 joints: 8 ft. 3 in. long and 3 joints: 9 ft. 2 in. long. No recementing is required on 1/4 of the job. How much recementing will need to be done?

ORDER OF OPERATION

When a mathematical problem requires a series of operations involving addition, subtraction, multiplication, or division, the order in which the operations are performed is important.

A series of individual additions, subtractions, or multiplications may be performed in any order. The numbers may be combined or grouped in whatever way is easiest to solve. For example, the following problems could be grouped as shown:

$$4 + 2 + 7 + 6 = 19 \quad \text{or} \quad 10 + 9 = 19$$

$$100 - 3 - 10 - 20 = 67 \quad \text{or} \quad 70 - 3 = 67$$

$$4 \times 2 \times 7 \times 5 = 280 \quad \text{or} \quad 40 \times 7 = 280$$

If division is part of the problem, the division must be worked or grouped in the order written. For example,

$$100 \div 10 \div 2 = 10 \div 2 = 5$$

In a series of mixed operations, perform multiplication first, division next, and finally addition and subtraction. For example:

$$100 \div 4 \times 5 = 5 \quad \text{or} \quad 100 \div 20 = 5$$

$$60 - 25 \div 5 = 55 \quad \text{or} \quad 60 - 5 = 55$$

Now consider the following:

$$\begin{aligned} 60 - 25 \div 5 + 15 - 100 + 4 \times 10 \\ = 60 - 25 \div 5 + 15 - 100 + 40 \\ = 60 - 5 + 15 - 100 + 40 \\ = 55 + 15 - 100 + 40 \\ = 110 - 100 \\ = 10 \end{aligned}$$

PRACTICE PROBLEMS:

26. $(10 - 8) 2 =$

27. $(25 - 6) + 3 + (8 \times 4) =$

28. $17 - 5 + (3 \times 3) =$

29. $(5 \times 9) + 9 - (6 \times 8) =$

30. $(3 \times 4) - 16 \div 4 + (3 \times 3) - 11 =$

METRIC MEASUREMENT

English measurement is written with a number and a unit of measure. It shows “how many” or “how much.” Examples of English measurements are 1 pound (weight), 1 foot (length), and 1 quart (volume). Since this system is well known to all, it will not be covered in depth in this topic.

The Metric System is based on the meter as the basic unit of measurement for length and distance. Millimeters, centimeters and decimeters are smaller than the meter while dekameter, hectometer and kilometer are larger than a meter.

The metric system is based on the number 10. All the metric units are in multiple of 10. For example:

Millimeter = 1/1000 of a meter (0.001) It takes 1000 millimeters to make a meter

Centimeter = 1/100 of a meter (0.01) It takes 100 centimeters to make a meter

Decimeter = 1/10 of a meter (0.1) It takes 10 decimeter to make a meter

Meter = 1 meter

Dekameter = 10 meters

Hectometer = 100 meters

Kilometer = 1000 meters

The metric system is not difficult to learn, it is just different from the English system. Metrics should be learned as a “stand alone” system of measurement.

It is believed by many that metrics is not only easier to use than the English system of measurement but it is also much more accurate. Figure 6 contains several metric conversion charts to help you become familiar with metric measurements.

The metric system is based on multiple of 10. For example, 20 millimeters is equivalent to 2 centimeters and 145 millimeters is equivalent to 14.5 centimeters. By simply moving the decimal point, metric measurement can be found (see Figure 6 and 7).

For example, if meters need to be converted to decimeters, move the decimal point one space to the right and add a zero as shown:

$$12 \text{ meters} = 120 \text{ decimeters}$$

If 6 meters needs to be converted to centimeters, move the decimal point two spaces to the right and add two zeros as shown:

$$6 \text{ meters} = 600 \text{ centimeters.}$$

The reverse is true when converting meters to dekameters, hectometers and kilometers.

Figures 6 & 7- Metric Conversion Charts

Length (Basic Unit Is Meter)

1 millimeter (mm)	=		=	.001	m
1 centimeter (cm)	=	10 millimeters	=	.01	m
1 decimeter (dm)	=	10 centimeters	=	.1	m
1 METER (m)	=	10 decimeters	=	1	m
1 dekameter(dam)	=	10 meters	=	10	m
1 hectometer (hm)	=	10 dekameters	=	100	m
1 kilometer (km)	=	10 hectometers	=	1000	m

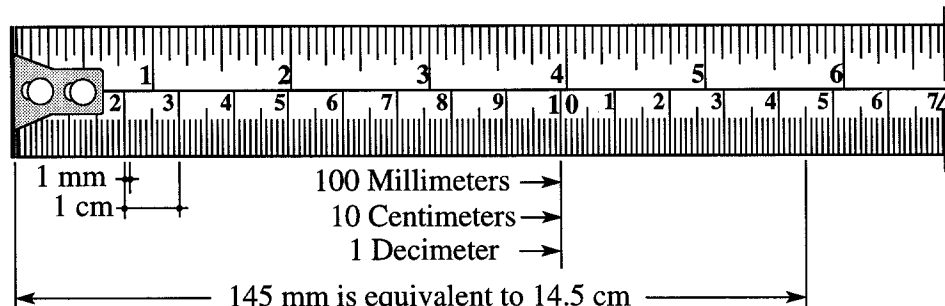
English - Metric Conversions

1 inch	=	2.54 centimeters
1 foot	=	0.3048 meter
1 yard	=	0.9144 meter
1 mile	=	1.609 kilometers
1 quart	=	0.946 liter
1 gallon	=	3.785 liters
1 ounce	=	28.35 grams
1 pound	=	453.59 grams

Metric - English Conversions

1 centimeter	=	0.3937 inch
1 meter	=	3.281 feet
1 meter	=	1.094 yards
1 kilometer	=	0.6214 mile
1 liter	=	1.057 quart
1 liter	=	0.2642 gallon
1 gram	=	0.0353 ounce
1 gram	=	0.0022 pound

ENGLISH MEASUREMENT



METRIC MEASUREMENT

Figures 6 & 7 - Metric Conversion Charts (continued)

CONVERSION CHART

Inch, Decimal, and MM Equivalents

INCH	DECIMAL	MM	INCH	DECIMAL	MM	INCH	DECIMAL	MM
$\frac{1}{64}$.015625	0.3969	$\frac{25}{64}$.390625	9.9219	$\frac{49}{64}$.765625	19.4469
$\frac{1}{32}$.03125	0.7938	$\frac{13}{32}$.40625	10.3188	$\frac{25}{32}$.78125	19.8438
$\frac{3}{64}$.046875	1.1906	$\frac{27}{64}$.421875	10.7156	$\frac{51}{64}$.796875	20.2406
$\frac{1}{16}$.0625	1.5875	$\frac{7}{16}$.4375	11.1125	$\frac{13}{16}$.8125	20.6375
$\frac{5}{64}$.078125	1.9844	$\frac{29}{64}$.453125	11.5094	$\frac{53}{64}$.828125	21.0344
$\frac{3}{32}$.09375	2.3812	$\frac{15}{32}$.46875	11.9062	$\frac{27}{32}$.84375	21.4312
$\frac{7}{64}$.109375	2.7781	$\frac{31}{64}$.484375	12.3031	$\frac{55}{64}$.859375	21.8281
$\frac{1}{8}$.1250	3.1750	$\frac{1}{2}$.5000	12.7000	$\frac{7}{8}$.8750	22.2250
$\frac{9}{64}$.140625	3.5719	$\frac{33}{64}$.515625	13.0969	$\frac{57}{64}$.890625	22.6219
$\frac{5}{32}$.15625	3.9688	$\frac{17}{32}$.53125	13.4938	$\frac{29}{32}$.90625	23.0188
$\frac{11}{64}$.171875	4.3656	$\frac{35}{64}$.546875	13.8906	$\frac{59}{64}$.921875	23.4156
$\frac{3}{16}$.1875	4.7625	$\frac{9}{16}$.5625	14.2875	$\frac{15}{16}$.9375	23.8125
$\frac{13}{64}$.203125	5.1594	$\frac{37}{64}$.578125	14.6844	$\frac{61}{64}$.953125	24.2094
$\frac{7}{32}$.21875	5.5562	$\frac{19}{32}$.59375	15.0812	$\frac{31}{32}$.96875	24.6062
$\frac{15}{64}$.234375	5.9531	$\frac{39}{64}$.609375	15.4781	$\frac{63}{64}$.984375	25.0031
$\frac{1}{4}$.2500	6.3500	$\frac{3}{8}$.6250	15.8750	1	1.0000	25.4000
$\frac{17}{64}$.265625	6.7469	$\frac{41}{64}$.640625	16.2719	1 INCH = 25.4000 MILLIMETERS 1 MILLIMETER = .039370113		
$\frac{9}{32}$.28125	7.1438	$\frac{21}{32}$.65625	16.6688			
$\frac{19}{64}$.296875	7.5406	$\frac{43}{64}$.671875	17.0656			
$\frac{5}{16}$.3125	7.9375	$\frac{11}{16}$.6875	17.4625			
$\frac{21}{64}$.328125	8.3344	$\frac{45}{64}$.703125	17.8594			
$\frac{11}{32}$.34375	8.7312	$\frac{23}{32}$.71875	18.2562			
$\frac{23}{64}$.359375	9.1281	$\frac{47}{64}$.734375	18.6531			
$\frac{3}{8}$.3750	9.5250	$\frac{3}{4}$.7500	19.0500			

PRACTICE PROBLEMS

PRACTICE PROBLEMS

31. 30 m = _____ decimeters

32. A board 2.4 meters long is to be cut into 5 equal parts. How many centimeters long will each piece be?

33. What is the measure of 8 meters + 45 cm in meters?

34. What is the measure of 35 meters and 2.4 cm in cms?

35. Convert the following
 - a. 3 dm = _____ mm
 - b. 1 m = _____ dm
 - c. 6 cm = _____ mm
 - d. 5 m = _____ cm

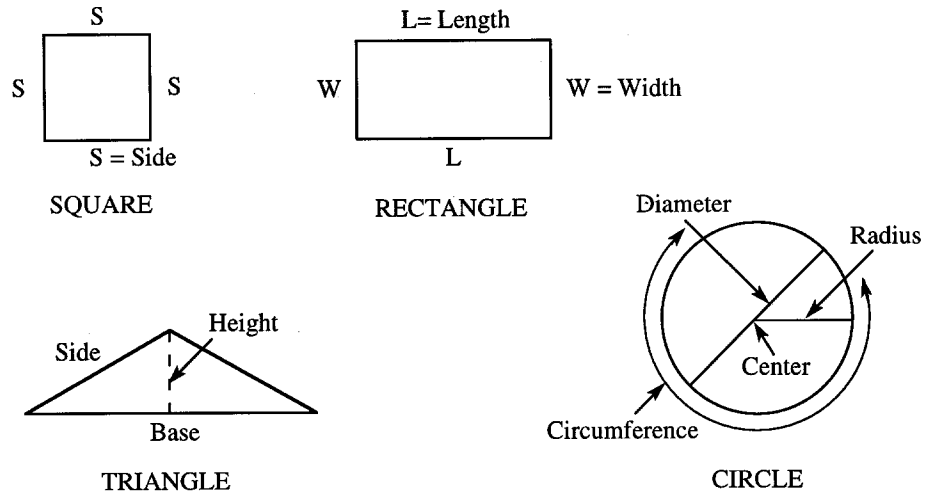
PERIMETER AND CIRCUMFERENCE

Circles, rectangles, triangles and squares are four examples of geometric figures or shapes (see Figure 8). The distance around any geometric figure is called the perimeter.

- The perimeter of a circle is called the circumference.
- The radius of the circle is the distance from the center to any point on the circle.
- The diameter is twice the radius.

The perimeter or circumference can be thought of as the fence around a figure. The perimeter or circumference is the length of that fence.

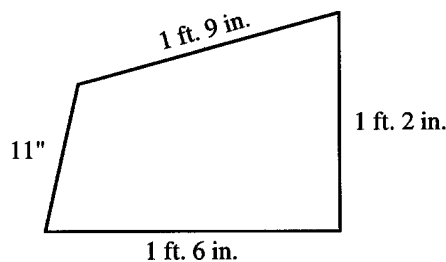
Figure 8 - Geometric Figures



GEOMETRIC FIGURES

The perimeter of any figure in which all of the sides are straight is the sum of the lengths of the sides. For example, find the perimeter of Figure 9. P (the perimeter) is the sum of the lengths of the four sides:

Figure 9 - Finding the Perimeter



1 ft.	9 in.
1 ft.	2 in.
1 ft.	6 in.
	11 in.
3 ft.	28 in.

28 in. = 2 ft. 4 in.

P = 5 ft. 4 in.

FINDING THE PERIMETER

There are formulas for finding the perimeter of squares, rectangles, and circles and for the length of a semicircle. To find the perimeter of a geometric figure that is not one of these mentioned, add the lengths of the sides.

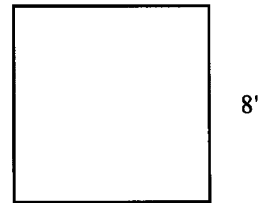
The formula for finding the perimeters are as follows:

Square

If "P" is the perimeter and "S" is the length of one side, then the formula is:

$$P = 4 \times S.$$

$$P = 4 (8 \text{ inches}) \\ = 32 \text{ inches}$$

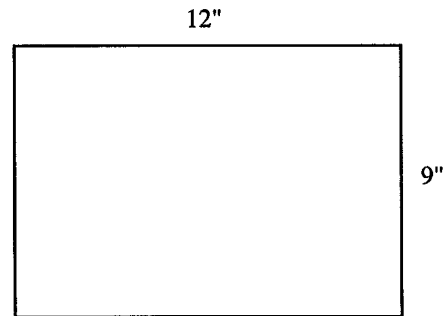


SQUARE

Rectangle

If "P" is the perimeter, "L" is the length and "W" is the width, then the formula is: $P = 2 \times L + 2 \times W$

$$P = 2 (9 \text{ in.}) + 2 (12 \text{ in.}) \\ = 18 \text{ in.} + 24 \text{ in.} \\ = 42 \text{ in.}$$

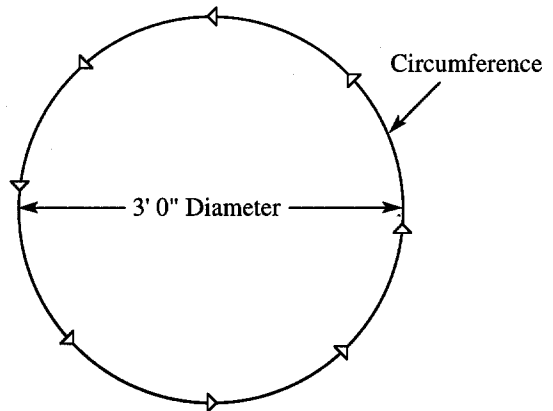


RECTANGLE

Circle

If "C" is the circumference and "D" is the diameter, then the formula is (π is read "pi" and its value is approximately 3.1416): $C = \pi \times D$ or $2 \times \pi \times R$

$$C = 3.1416 \times 3 \text{ ft} \\ \text{or } 2 \times 3.1416 \times 1.5 \text{ ft} \\ = 9.4248 \text{ ft}$$



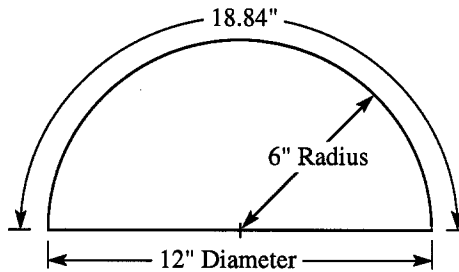
CIRCLE

Semicircle

If "L" is the length and "R" is the radius, then the formula is: $L = \pi \times R$ or $\pi \times D \div 2$

$$L = 3.1416 \times 6 \text{ or } \pi \times D \div 2$$

$$= 18.8496 \text{ in.}$$



SEMICIRCLE

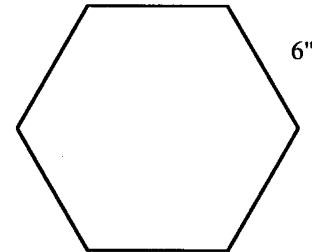
PRACTICE PROBLEMS

36. Find the perimeter of a square that is 20 inches on each side.

P =

37. Find the perimeter of the following hexagon.

P =



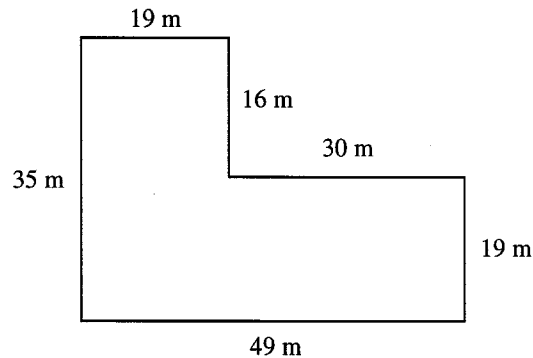
38. Find the circumference of a circle that has a radius of 12 cm

P =

39. Find the circumference of a triangle that has sides of 7, 8 and 9 inches.

40. Find the perimeter of the following figure:

P =



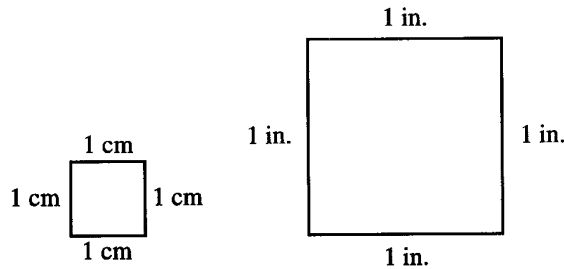
41. What is the circumference of a circle with a two foot diameter.

FINDING THE AREA OF SHAPES

While “perimeter” refers to the distance around a figure (like building a fence around an object), “area” refers to the measurement of a surface.

In other words, “area” can be thought of as laying tile on a floor. The area is the number of square (tiles) it takes to cover the floor. To find the surface area of a shape means to determine how many squares are contained within that figure. Each square is the same size and is called a square unit. Two examples of surface measures are shown in Figure 10.

Figure 10 -



FINDING THE SURFACE AREA

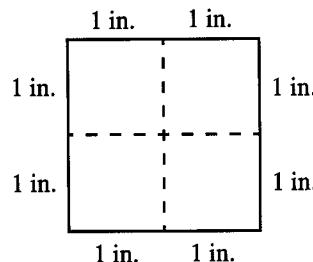
The unit of measure on the right is called a “square inch” since the square is one inch on each side. The “square inch” measures the surface that is contained within the square.

The unit of measure on the left is called a “square centimeter” since the square is one centimeter on each side. The “square centimeter” also measures the surface that is contained within the square.

There are other units of surface measure such as square foot, square yards, square miles, square meters, and square kilometers.

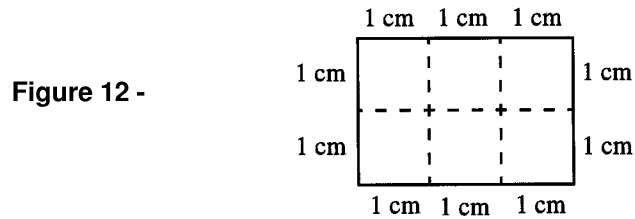
Now consider a square that is 2 inches on each side (see Figure 11). The square can be divided into four squares, each of which is one inch on a side. This shows that the area of the square is four square inches. The area is the square of the length of a side: $\text{area} = 2 \text{ inches} \times 2 \text{ inches}$. In other words, the area of a square can be found by squaring the length of one of its equal sides or $A = s(2)$.

Figure 11 -



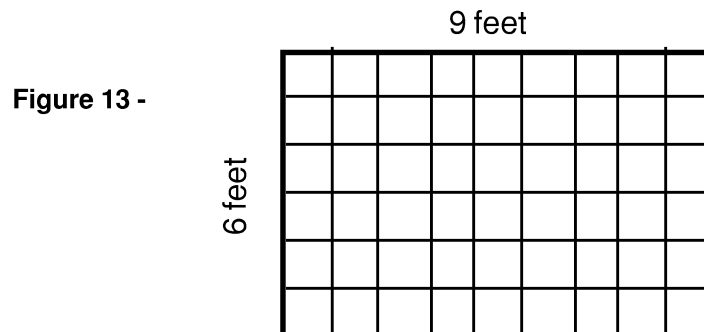
FINDING SURFACE AREA

Consider a rectangle that has a length of 3 centimeters and a width of 2 centimeters (see Figure 12). The rectangle can be divided into six squares, each of which is one centimeter on a side. This tells us that the area of the rectangle is 6 cm. If we multiply the length (3 cm) times the width (2 cm) we have $(3 \text{ cm}) (2 \text{ cm}) = 6 \text{ cm}$. The area of a rectangle can be found by multiplying the length times the width or $A = L \times W$.



FINDING SURFACE AREA IN CENTIMETERS

Consider a rectangle that has a length of 9 feet and a width of 6 feet. The rectangle can be divided into 54 squares, each of which is 1 foot on a side. This tells us that the area of the rectangle is 54 feet, or $A = L \times W$.



FINDING SURFACE AREA IN SQUARE FEET

Square Yards

Using the rectangle in Figure 13 to find the amount of square yards contained within it the following formula is used. There are 9 square feet in one square yard. Take the length of 9 feet times the width of 6 feet and dividing the answer by 9 will give the square yardage contained in Figure 13.

The following formulas are used to find the area of a parallelogram, a triangle, a trapezoid, and a circle (see Figure 14):

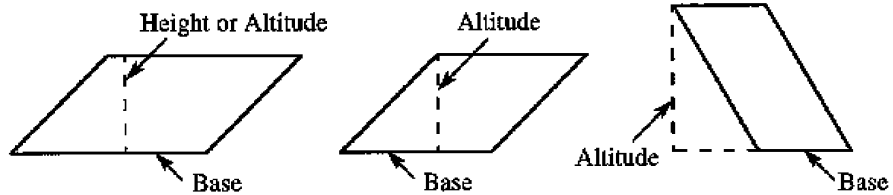
The area of a parallelogram: $A = \text{base} \times \text{height}$

The area of a triangle: $A = \frac{\text{base} \times \text{height}}{2}$

The area of a trapezoid: $A = \frac{(B_1 + B_2) \times H}{2}$

The area of a circle: $A = \pi R^2$

Figure 14 -



PARALLELOGRAMS



TRAPEZOIDS

SURFACE AREA OF PARALLELOGRAMS AND TRAPEZOIDS

Parallelogram

To find the area of a parallelogram with a base of 1 ft and a height of 4 inches, the following steps should be performed:

$$\begin{aligned} A &= b \times h \\ &= (1 \text{ ft.}) (4 \text{ in.}) \text{ must be converted to inches} \\ &= (12 \text{ in.}) (4 \text{ in.}) \\ &= 48 \text{ inches} \end{aligned}$$

Triangle

Using the triangle formula ($A = \frac{1}{2} \times b \times h$), find the area of a triangle with $b = 14.2 \text{ cm}$ and $h = 7 \text{ cm}$.

$$\begin{aligned} A &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 14.2 \text{ cm} \times 7 \text{ cm} \\ &= 49.7 \text{ square centimeter} \end{aligned}$$

Trapezoid

Using the trapezoid formula ($A = \frac{1}{2} (b_1 + b_2) \times h$), find the area of a trapezoid with $b_1 = 17$ ft., $b_2 = 13$ ft., and $h = 7$ ft.

$$\begin{aligned} A &= \frac{1}{2} (b_1 + b_2) \times h \\ &= \frac{1}{2} (17 \text{ ft} + 13 \text{ ft}) \times 7 \\ &= \frac{1}{2} (30 \text{ ft}) (7 \text{ ft}) \\ &= 105 \text{ sq. ft} \end{aligned}$$

Circle

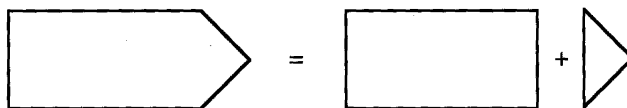
Using the formula for finding the area of a circle ($A = p \times R^2$), find the area of a circle with $R = 8$ cm (let $p = 3.1416$)

$$\begin{aligned} A &= p \times R^2 \\ &= (3.1416) (8 \text{ cm})^2 \\ &= (3.1416) (64 \text{ sq cm}) \\ &= 201.0624 \text{ sq cm} \end{aligned}$$

Some figures can be divided into two or more of the common shapes. The sum of the areas of each is the area of the entire surface.

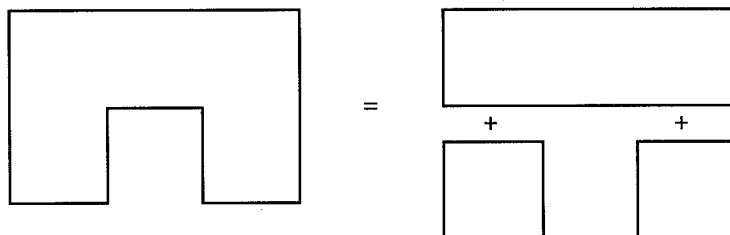
For example (see Figure 15), the figure on the left can be divided into a rectangle with a triangle attached. We can find the area of the rectangle and the area of the triangle. The sum of these areas is the area of the entire surface.

Figure 15 -



In the next example (see Figure 16), the figure on the left can be divided into three rectangles. Find the area of each rectangle. The sum of these areas is the area of the entire surface.

Figure 16 -



PRACTICE PROBLEMS

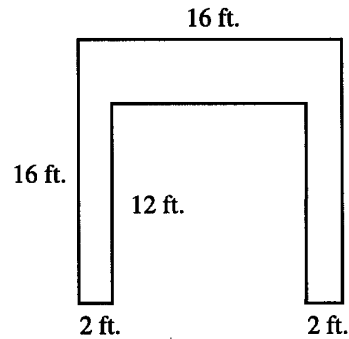
To find the area of a geometric figure that is a combination of two or more common figures:

1. Divide the figure into common geometric figures for which the information is given or attach or square up a region and then divide into common geometric figures for which you can find the area.
2. Find the area of each of the common figures.
3. Find the sum or difference of those areas.

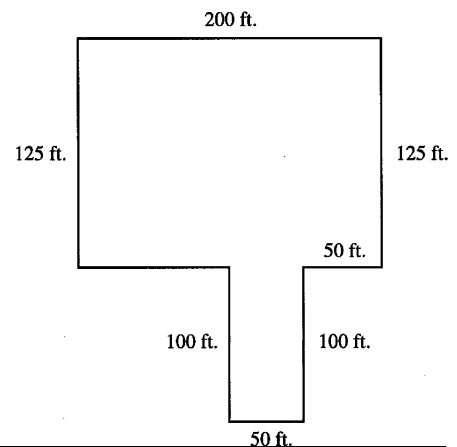
PRACTICE PROBLEMS:

42. Find the area of a square that is 4 inches on each side.
43. Find the area of a square that is 11 cm on each side.
44. Find the area of a rectangle with $L = 12$ m and $W = 8$ m.

45. Find the area of the following figure:



46. Find the area of the following figure:



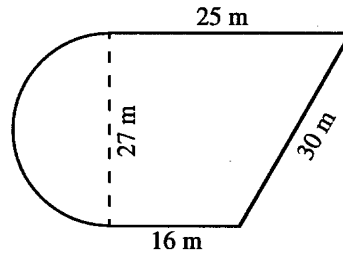
REVIEW

NAME:

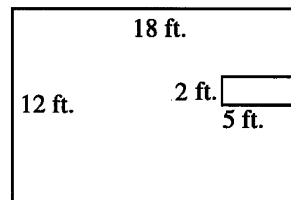
DATE:

1. What is meant by the term “place value?”
2. What is meant by the term “carrying” a number?
3. What is the basic unit of measurement for length and distance in the metric system?
4. A centimeter is _____ of a meter.
5. A decimeter is _____ of a meter.
6. What is the definition of “perimeter”?
7. What is the perimeter of a circle called?
8. What is the definition of “radius?”
9. Find the perimeter of a triangle that has sides of 7 in, 8 in, and 9 in.?
10. Find the perimeter of a square that is 18 inches on each side?

11. Find the perimeter of the following figure (let $p = 3.1416$)



12. Find the perimeter of the following figure:



13. Find the area of a trapezoid with bases of 51 m and 36 m and height of 12 m.

14. $460 \times 15 =$

15. $180 \div 5 =$

16. Six lites of $\frac{1}{2}$ " glass, 72" x 120", that weigh 6 lbs. per sq. ft and 4 lites of $\frac{1}{4}$ " glass, 48" x 96", that weigh 3 lbs per sq. ft. are packed in a case that weighs 820 lbs. What is the total weight of the case and its contents in pounds?

17. What is the total length of the following pieces of metal: 1 piece 1 ft. 10 in., 1 piece 10ft. 2in., 1 piece 16ft. 3 in. and 1 piece 8 ft. 1 in. long?

18. A glass shop receives an order to replace the tops on 6 showcases. Each of these showcases requires a piece of green felt 2 inches wide and 6 feet 3 inches long under the rear edge of the glass only. How many square inches of green felt will be needed to do the entire job?

19. Three identical metal frames are needed to complete a certain job. The following pieces of metal extrusion are required to make these frames; 8 pieces 10 ft. 6 in. long, 9 pieces 8ft 4in. long and 3 pieces 3ft. 9 in. long. How many inches of the metal will be required for each frame?
20. The following pieces of material were cut from a stock of 10 lengths, each 21 ft. long, 2 pieces 4ft. long, 3 pieces 6 ft. 4 inches long and 4 pieces 54 inches long. How many feet of the stock material remained after the pieces were cut?
21. The following area is 7 feet by 4 feet. What is the total yardage of the area.

